Dynamical effects of counter-rotating couplings on interference between driving and dissipation

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We use an analytical method to study the dissipative dynamics of a two-level system (TLS) under a harmonic driving. The method is based on a combination of the unitary transformation and Born–Markov master-equation approach. Our main aim is to clarify the effects of counter-rotating (CR) terms of both the driving and TLS-bath (dissipative) coupling on the dynamics, in comparison with the rotating-wave results of different schemes, i.e., the well-known traditional rotating-wave approximation method, and two particular methods: one just takes into account the effects of the driving CR terms and the other the effects of the dissipative CR terms, which are derived from our general treatment. Our main results are as follows: (i) by calculating the time-dependent population difference and coherence, in the case of resonant strong driving, we demonstrate that the CR terms of both the driving and dissipative coupling play an important role in the relaxation and dephasing processes, and also the properties of the steady state; (ii) in the case of largely detuned driving, we find that the CR terms of the dissipative coupling become negligible while those of the driving contribute dominant modifications to the time evolution and steady state. Moreover, we examine the influence of the dissipation on coherent destruction of tunneling under a largely detuned strong driving. We show that an almost complete suppression of the tunneling can be achieved for a relatively long time; (iii) under certain conditions, we find numerical equivalence between one of two particular methods and the Floquet–Born–Markov approach based on exact numerical treatment of the Floquet Hamiltonian. It turns out that our method is more simple and efficient than the Floquet–Born–Markov approach for both analytical and numerical calculations. By the general comparison of different treatments we demonstrate the dynamical effects of CR terms of both the driving and the dissipative coupling on the coherence and population difference.

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I. INTRODUCTION

The driven spin-boson model (SBM) has been widely studied in experiment and theory, which is related to various physical and chemical processes [1–4]. The model describes the physics of a two-level system (TLS) driven by an external force and coupled to a dissipative bosonic bath. The main interest of the study is to understand how the interplay between the driving and the dissipation influences time evolution and decoherence of the TLS. Theoretically, the Hamiltonian of the model in the localized representation reads ($\hbar = 1$)

$$\begin{aligned} H(t) &= -\frac{1}{2}[\Delta\sigma_x + A\cos(\omega_L t)\sigma_z] \\ &\quad + \sum_k \omega_k b_k^\dagger b_k + \frac{1}{2}\sum_k g_k(b_k^\dagger + b_k)\sigma_z \\ &= -\frac{1}{2}\Delta\sigma_x - \frac{A}{4}(e^{i\omega_L t}\sigma_- + e^{-i\omega_L t}\sigma_+) + \sum_k \omega_k b_k^\dagger b_k \\ &\quad + \frac{1}{2}\sum_k g_k(b_k^\dagger\sigma_- + b_k\sigma_+) + H_{\mathrm{CR1}}(t) + H_{\mathrm{CR2}}, \quad (1) \end{aligned}$$

$$H_{\mathrm{CR1}}(t) = -\frac{A}{4}(e^{-i\omega_L t}\sigma_- + e^{i\omega_L t}\sigma_+), \quad (2)$$

$$H_{\mathrm{CR2}} = \frac{1}{2}\sum_k g_k(b_k^\dagger\sigma_+ + b_k\sigma_-), \quad (3)$$

where $\sigma_\mu(\mu = x,y,z)$ is the μ-component Pauli matrix and $\sigma_\pm = (\sigma_z \pm i\sigma_y)/2$. Δ is the bare tunneling and $A\cos(\omega_L t)$ is a time-dependent driving force of amplitude A and frequency ω_L. b_k ($b_k^\dagger$) is the annihilation (creation) operator of the kth boson mode with frequency ω_k, and g_k is the coupling strength between the TLS and kth mode of the bath. Besides, we consider that the effect of the bosonic bath is characterized by the Ohmic spectral density $G(\omega) = \sum_k g_k^2\delta(\omega - \omega_k) = 2\alpha\omega\theta(\omega_c - \omega)$ in which α is the dimensionless coupling constant, θ is the usual step function, and ω_c is the cutoff frequency. $H_{\mathrm{CR1}}(t)$ and H_{CR2} are the counter-rotating (CR) terms of the driving and dissipative coupling, respectively. When omitting $H_{\mathrm{CR1}}(t) + H_{\mathrm{CR2}}$, one can transform the Hamiltonian into a time-independent form, which can be treated based on the Born–Markov master-equation approach [5,6]. Usually, the neglect of CR terms is called the rotating-wave approximation (RWA), which is valid in the regime of weak driving and weak damping. The neglect of $H_{\mathrm{CR1}}(t)$ is also called the Rabi-RWA. In the present paper, we propose a novel unitary transformation to transform the Hamiltonian with $H_{\mathrm{CR1}}(t) + H_{\mathrm{CR2}}$ into a RWA-like form and at the same time physically take into account the influence of the CR terms on the time evolution of the system in a wide parameter space beyond weak driving and weak damping.

In spite of the simple form of the SBM Hamiltonian [Eq. (1)], it is a nontrivial task to obtain the analytical solution concerning the dynamical evolution. Various approximate methods have been applied to study dissipative dynamics in such a system; for instance, the traditional quantum optics approach [6–8], the path integral approach

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[2,9–12], the noninteracting blip approximation (NIBA) [1,2], the Born–Markov master-equation approach [12–15], the nonperturbative stochastic method [16], etc. Within these methods, the NIBA provides a good approximate description for strong damping and intermediate-to-high bath temperatures. In contrast, the approaches based on the Born–Markov approximation are preferred for weak damping and low temperature. It has been demonstrated that the time-local Born–Markov master equation is numerically equivalent to the path integral approach in the regime of weak damping and low temperature [12]. In general, the time-local master equations are not suited for analytical calculation and one usually uses the stationary Born–Markov master equation such as the works in Refs. [6,14,15], which can be conveniently derived.

To take into account the CR terms of the driving, one usually combines the Floquet theory with the Born–Markov master-equation approach, which leads to the so-called Floquet–Born–Markov (FBM) master equation [3,14]. One can find the general procedure of the approach and its applications in Refs. [3,14,17–20]. This method requires diagonalizing the Floquet Hamiltonian and invoking a moderate RWA (MRWA) to remove the explicit time-dependence in the corresponding master equation [3]. The Floquet Hamiltonian is usually easily diagonalized by a numerically exact treatment but hardly by an analytically exact calculation [3,14]. Therefore, one usually applies the perturbation theory in the amplitude A for weak driving [21] or in the tunneling Δ for strong driving [14]. The perturbative method cannot uniformly treat the driving from weak to strong strength. For instance, in the limit $A/\omega_L \gg 1$, the validity of the FBM approach based on von Vleck perturbation theory in the tunneling Δ requires $A \gg \Delta$ [14]. Besides, the role of the MRWA should be carefully examined.

In our previous work, we considered H_{CR2} and took into account the CR terms of the dissipative coupling by a unitary transformation [22]. The transformation allows us to keep the same mathematical simplicity of the formalism as for the traditional RWA approach with dropping $H_{\mathrm{CR1}}(t) + H_{\mathrm{CR2}}$. Although we avoid neglecting H_{CR2} in Ref. [22], we have employed the Rabi-RWA in which $H_{\mathrm{CR1}}(t)$ is omitted. Thus, the method in Ref. [22] is not suited for discussing moderately strong driving situations. To overcome this limitation, here, we develop an extended approach based on a novel unitary transformation. The unitary transformation is applied to generate an effective Hamiltonian involving the effects of all CR terms, i.e., $H_{\mathrm{CR1}}(t) + H_{\mathrm{CR2}}$. More importantly, the effective Hamiltonian keeps the RWA-like form with modified parameters. In this sense, the mathematical structure of the method is as simple as the traditional RWA approach. On the other hand, our general treatment may lead to two particular cases: one is the case of Ref. [22] which does not consider the effects of the driving CR terms, the other is that which does not take into account the effects of the dissipative CR terms. Besides, we find that the treatment based on the second particular case can reproduce numerical results obtained by the FBM master equation under certain conditions, which helps us see clearly the role of the MRWA. Moreover, our method is analytically simple and enables us to uniformly treat weak to moderately strong driving, as compared with the FMB approach based on perturbation theory.

The structure of the paper is as follows: In Sec. II, we mainly introduce our general treatment based on the unitary transformation as well as two particular treatments derived from the general one. Besides, we discuss the validity of our general treatment. In Sec. III, we derive the master equation for the reduced density matrix governed by the effective Hamiltonian we constructed and give its analytical solutions. In Sec. IV, by calculating population difference and coherence, we examine numerically and analyze the effects of the CR terms on the time evolution and steady state of the system. In addition, we reveal the relation between analytical solutions derived by a particular treatment and those of the FBM approach based on the exact numerical treatment of the Floquet Hamiltonian. In the last section, we give a summary.

II. UNITARY TRANSFORMATION FORMALISM

A. Traditional rotating-wave approximation approach

We first review briefly the method based on the RWA. In the weak-damping and weak-driving regime, it is convenient to treat the issue using the traditional RWA approach in quantum optics, which is based on the neglect of $H_{\mathrm{CR1}}(t)$ and H_{CR2}. After the approximation, the Hamiltonian possesses the form

$$H^{\mathrm{RWA}}(t) = -\frac{1}{2}\left[\Delta\sigma_x + \frac{A}{2}(e^{i\omega_L t}\sigma_- + e^{-i\omega_L t}\sigma_+)\right] + \sum_k \omega_k b_k^\dagger b_k + \frac{1}{2}\sum_k g_k(b_k^\dagger\sigma_- + b_k\sigma_+). \quad (4)$$

To proceed, the usual way is to transform the explicitly time-dependent Hamiltonian into a time-independent Hamiltonian with the unitary rotating operation

$$R(t) = \exp\left[i\omega_L t\left(-\frac{1}{2}\sigma_x + \sum_k b_k^\dagger b_k\right)\right], \quad (5)$$

which leads to the time-independent form

$$\tilde{H}^{\mathrm{RWA}} = -\frac{1}{2}\left(\delta_0\sigma_x + \frac{A}{2}\sigma_z\right) + \sum_k(\omega_k - \omega_L)b_k^\dagger b_k + \frac{1}{2}\sum_k g_k(b_k^\dagger\sigma_- + b_k\sigma_+), \quad (6)$$

where $\delta_0 = \Delta - \omega_L$ is the detuning. Starting from this Hamiltonian and using the Born–Markov approximation [4,23], one can easily derive the master equation for the reduced density matrix of the TLS. Although the approach is mathematically simple, it is at the expense of dropping $H_{\mathrm{CR1}}(t) + H_{\mathrm{CR2}}$. Thus, it is only valid to discuss on-resonance or near-resonance physics in the regimes of weak driving and weak damping. In the following, we introduce a method to derive an effective Hamiltonian taking the same mathematical form as $\tilde{H}^{\mathrm{RWA}}$ and properly involving the effects of the CR terms of both the driving and dissipative coupling.

B. General formalism and particular cases

To study the driven SBM physics from weak to strong driving strength and beyond the weak-damping limit, we propose a method based on the unitary transformation $H'(t) = e^{S(t)}H(t)e^{-S(t)} - ie^{S(t)}\frac{d}{dt}e^{-S(t)}$. The generator of the unitary transformation is

$$S(t) = \left[-i\frac{A}{2\omega_L}\zeta\sin(\omega_L t) + \sum_k \frac{g_k}{2\omega_k}\xi_k(b_k^\dagger - b_k)\right]\sigma_z, \quad (7)$$

where the parameters $\zeta \in (0,1)$ and $\xi_k \in (0,1)$ will be determined later. This generator is a combination of the two generators in Refs. [22,24]. When $\xi_k = 0$, it becomes the generator used for taking account of the effects of the CR terms of the driving [24]. When $\zeta = 0$, the generator (7) changes into the generator of treating the CR terms of dissipative coupling [22,25,26]. Setting $X_1 = \sum_k \frac{g_k}{\omega_k}\xi_k(b_k^\dagger - b_k)$, we perform the unitary transformation and obtain the resulting Hamiltonian,

$$H'(t) = H_0'(t) + H_1' + H_2'(t), \quad (8)$$

$$\begin{aligned} H_0'(t) = &-\frac{1}{2}J_0\left(\frac{A}{\omega_L}\zeta\right)\eta\Delta\sigma_x - J_1\left(\frac{A}{\omega_L}\zeta\right)\eta\Delta\sin(\omega_L t)\sigma_y \\ &-\frac{1}{2}A(1-\zeta)\cos(\omega_L t)\sigma_z \\ &+\sum_k \omega_k b_k^\dagger b_k + \sum_k \frac{g_k^2}{4\omega_k}\xi_k(\xi_k - 2), \end{aligned} \quad (9)$$

$$\begin{aligned} H_1' = &-\frac{1}{2}J_0\left(\frac{A}{\omega_L}\zeta\right)\eta\Delta i\sigma_y \sum_k \frac{g_k}{\omega_k}\xi_k(b_k^\dagger - b_k) \\ &+\frac{1}{2}\sum_k g_k(1-\xi_k)(b_k^\dagger + b_k)\sigma_z, \end{aligned} \quad (10)$$

$$\begin{aligned} H_2'(t) = &-\frac{1}{2}J_0\left(\frac{A}{\omega_L}\zeta\right)\Delta(\cosh X_1 - \eta)\sigma_x \\ &-\frac{1}{2}J_0\left(\frac{A}{\omega_L}\zeta\right)\Delta(\sinh X_1 - \eta X_1)i\sigma_y \\ &- J_1\left(\frac{A}{\omega_L}\zeta\right)\Delta(\cosh X_1 - \eta)\sin(\omega_L t)\sigma_y \\ &+ iJ_1\left(\frac{A}{\omega_L}\zeta\right)\Delta\sigma_x\sin(\omega_L t)\sinh X_1 \\ &- \Delta\Big(\sigma_x\cosh X_1 + i\sigma_y\sinh X_1\Big) \\ &\times \sum_{n=1}^{\infty} J_{2n}\left(\frac{A}{\omega_L}\zeta\right)\cos(2n\omega_L t) \\ &- \Delta(\sigma_y\cosh X_1 - i\sigma_x\sinh X_1) \\ &\times \sum_{n=1}^{\infty} J_{2n+1}\left(\frac{A}{\omega_L}\zeta\right)\sin[(2n+1)\omega_L t], \end{aligned} \quad (11)$$

where η is determined by

$$\eta = \langle\{0_k\}|\cosh X_1|\{0_k\}\rangle = \exp\left[-\sum_k \frac{1}{2}\left(\frac{g_k}{\omega_k}\xi_k\right)^2\right], \quad (12)$$

with $|\{0_k\}\rangle$ being the vacuum state of the bosonic bath. $J_n(z)$ is the nth-order Bessel function of the first kind. When deriving the above equations, we used the identity $\exp(iz\sin\alpha) = \sum_{n=-\infty}^{\infty} J_n(z)e^{in\alpha}$ [27].

As shown in Eq. (8), we divided the transformed Hamiltonian into three parts according to harmonic oscillation frequency $n\omega_L$ and the order-of-coupling strength g_k. Except for the constant term, the first part $H_0'(t)$ consists of terms that satisfy two conditions: harmonic oscillation frequency $n\omega_L$ with $n = 0,1$ and zeroth order in g_k. If ζ is determined by

$$J_1\left(\frac{A}{\omega_L}\zeta\right)\eta\Delta = \frac{1}{2}A(1-\zeta) \equiv \frac{\tilde{A}}{4}, \quad (13)$$

$H_0'(t)$ can be rewritten as (the constant term is neglected)

$$\begin{aligned} H_0'(t) = &-\frac{1}{2}J_0\left(\frac{A}{\omega_L}\zeta\right)\eta\Delta\sigma_x - \frac{\tilde{A}}{4}(\sigma_+ e^{-i\omega_L t} + \sigma_- e^{i\omega_L t}) \\ &+ \sum_k \omega_k b_k^\dagger b_k. \end{aligned} \quad (14)$$

Obviously, $H_0'(t)$ can be solved exactly and serve as a free Hamiltonian.

The second part H_1' consists of the terms of order g_k and harmonic oscillation frequency $n\omega_L = 0$ and will serve as the perturbation Hamiltonian in the following treatment. If ξ_k is set by

$$\xi_k = \frac{\omega_k}{\omega_k + J_0\left(\frac{A}{\omega_L}\zeta\right)\eta\Delta}, \quad (15)$$

H_1' takes the RWA form

$$H_1' = \frac{1}{2}\sum_k \tilde{g}_k(b_k^\dagger\sigma_- + b_k\sigma_+), \quad (16)$$

with a modified coupling strength

$$\tilde{g}_k = g_k\frac{2J_0\left(\frac{A}{\omega_L}\zeta\right)\eta\Delta}{\omega_k + J_0\left(\frac{A}{\omega_L}\zeta\right)\eta\Delta}. \quad (17)$$

The third part $H_2'(t)$ can be omitted under certain conditions in spite of its complex form. Notice that the first two lines of $H_2'(t)$ represents the processes related to the multiboson nondiagonal transitions and their contributions to the physical quantities are of order g_k^4 and higher, which are negligible for low-temperature and moderately weak dissipative regimes. On the other hand, the remaining time-dependent terms are multiplied by $J_1(\frac{A}{\omega_L}\zeta)(\cosh X_1 - \eta)$, $J_n(\frac{A}{\omega_L}\zeta)\sinh X_1$ $(n \geqslant 1)$ and by $J_n(\frac{A}{\omega_L}\zeta)\cosh X_1$ $(n \geqslant 2)$, the contributions of which are negligible to the driven dynamics as compared with the time-dependent terms in $H_0'(t)$ due to the mixing of the Bessel functions and terms of order g_k or higher. Generally, the validity of the approximation of neglecting the higher-order Bessel functions $(n \geqslant 2)$ depends on the driving parameters A and ω_L. We will discuss the validity of our treatment in Sec. II C.

After omitting $H_2'(t)$, we arrive at our effective Hamiltonian $H^{\zeta\text{-}\xi_k\text{-TRWA}}(t) = H_0'(t) + H_1'$, where the superscript ζ-ξ_k-TRWA means that our effective Hamiltonian of the RWA-like form is obtained through the unitary transformation with the parameters ζ and ξ_k. Similarly, it is convenient to rotate it into a time-independent form by the same unitary rotating transformation $R(t)$. After the rotating operation, we readily obtain a time-independent Hamiltonian

$$\tilde{H}^{\zeta\text{-}\xi_k\text{-TRWA}} = -\frac{1}{2}\left(\tilde{\delta}\sigma_x + \frac{\tilde{A}}{2}\sigma_z\right) + \sum_k(\omega_k - \omega_L)b_k^\dagger b_k + \frac{1}{2}\sum_k \tilde{g}_k(b_k^\dagger\sigma_- + b_k\sigma_+), \tag{18}$$

where $\tilde{\delta} = J_0(\frac{A}{\omega_L}\zeta)\eta\Delta - \omega_L$. Comparing with $\tilde{H}^{\rm RWA}$, we find that in the present formalism the CR terms influence on the dynamics and stable properties through the modified quantities, i.e., $\tilde{\delta}$, $\tilde{A}$, and $\tilde{g}_k$. Besides, these quantities are determined by the driving parameters as well as the dissipative parameters, which reflects the interference between driving and dissipation. In particular, we find that the spectral density in our effective system is renormalized due to the modified dissipative coupling as

$$\tilde{G}(\omega) = \sum_k \tilde{g}_k^2\delta(\omega-\omega_k) = G(\omega)\left[\frac{2J_0\left(\frac{A}{\omega_L}\zeta\right)\eta\Delta}{\omega + J_0\left(\frac{A}{\omega_L}\zeta\right)\eta\Delta}\right]^2. \tag{19}$$

The key point of our method is clear, i.e., the unitary transformation is applied to construct an effective Hamiltonian that reduces the contributions of the omitted $H_2'(t)$ to make them as small as possible. Moreover, the effective Hamiltonian is simple enough for analytical calculations within the formalism of the Born–Markov master equation.

From our general treatment above, we give directly two unequal treatments by setting either ξ_k or ζ to zero. After the $\xi_k = 0$ ($\zeta = 0$) related transformation, we omit the CR terms of the dissipative coupling (the driving) to construct a RWA-like Hamiltonian. When making $\xi_k = 0$ in the unitary transformation, we obtain

$$H'(t) = H^{\zeta\text{-TRWA}}(t) + H_2'(t), \tag{20}$$

$$H^{\zeta\text{-TRWA}}(t) = -\frac{1}{2}J_0\left(\frac{A}{\omega_L}\zeta\right)\Delta\sigma_x - \frac{\tilde{A}}{4}(e^{i\omega_L t}\sigma_- + e^{-i\omega_L t}\sigma_+) + \sum_k \omega_k b_k^\dagger b_k + \frac{1}{2}\sum_k g_k(b_k^\dagger\sigma_- + b_k\sigma_+), \tag{21}$$

$$H_2'(t) = -\Delta\sum_{n=1}^{\infty}J_{2n}\left(\frac{A}{\omega_L}\zeta\right)\cos(2n\omega_L t)\sigma_x - \Delta\sum_{n=1}^{\infty}J_{2n+1}\left(\frac{A}{\omega_L}\zeta\right)\sin[(2n+1)\omega_L t]\sigma_y + \frac{1}{2}\sum_k g_k(b_k^\dagger\sigma_+ + b_k\sigma_-), \tag{22}$$

where the superscript ζ-TRWA represents that the CR terms of the driving is involved by the ζ-related unitary transformation and those of the dissipative coupling are neglected. The parameters ζ and $\tilde{A}$ are obtained from Eq. (13) with $\eta = 1$. Similarly, $H^{\zeta\text{-TRWA}}(t)$ can serve as effective Hamiltonian and $H_2'(t)$ will be omitted. Surprisingly, this resulting effective Hamiltonian can reproduce certain numerical results obtained by the FBM master equation based on exact numerical treatment of the Floquet Hamiltonian. We come back to discuss this in Sec. IV.

When $\zeta = 0$, the unitary transformation leads to another transformed Hamiltonian [22]

$$H'(t) = H^{\xi_k\text{-TRWA}}(t) + H_2'(t), \tag{23}$$

$$H^{\xi_k\text{-TRWA}}(t) = -\frac{1}{2}\eta\Delta\sigma_x - \frac{A}{4}(\sigma_+ e^{-i\omega_L t} + \sigma_- e^{i\omega_L t}) + \sum_k \omega_k b_k^\dagger b_k + \frac{1}{2}\sum_k \tilde{g}_k(b_k^\dagger\sigma_- + b_k\sigma_+), \tag{24}$$

$$H_2'(t) = -\frac{1}{2}\Delta\sigma_x(\cosh X_1 - \eta) - \frac{1}{2}\Delta i\sigma_y(\sinh X_1 - \eta X_1) - \frac{A}{4}(e^{-i\omega_L t}\sigma_- + e^{i\omega_L t}\sigma_+), \tag{25}$$

where the superscript ξ_k-TRWA represents that only the CR terms of the dissipative coupling are taken into account by ξ_k-related unitary transformation and those of the driving are not. Here, the parameters η, ξ_k, and $\tilde{g}_k$ for $H^{\xi_k\text{-TRWA}}(t)$ are determined by the corresponding Eqs. (12), (15), and (17) with $J_0(\frac{A}{\omega_L}\zeta)$ being replaced by 1 in the equations. This time the omitted term $H_2'(t)$ involves the CR terms of the driving and the terms of order g_k^2 or higher [see first line in Eq. (25)]. Comparing with $H^{\rm RWA}(t)$, we notice that the modifications resulting from the CR terms are included in the renormalized tunneling $\eta\Delta$ and the modified coupling strength $\tilde{g}_k$. In particular, the modified coupling strength leads to a renormalized spectral density

$$\tilde{G}(\omega) = \sum_k \tilde{g}_k^2\delta(\omega-\omega_k) = G(\omega)\left(\frac{2\eta\Delta}{\omega+\eta\Delta}\right)^2. \tag{26}$$

In order to show the intrinsic difference among the four effective Hamiltonians given above, we should pay attentions to the different approximations invoked in the four treatments. For the standard quantum optics approach, the omitted terms are $H_{\rm CR1}(t) + H_{\rm CR2}$. Therefore, the method is valid in the weak-driving and weak-damping regimes. For the effective Hamiltonian $H^{\zeta\text{-TRWA}}(t)$, we have directly dropped the CR terms $\frac{1}{2}\sum_k g_k(b_k^\dagger\sigma_+ + b_k\sigma_-)$ in the transformed Hamiltonian (20) and the terms with Bessel functions of second order and higher, and thus it is reasonably applied for weak to moderately strong driving in the weak dissipative regime, which is proved by our calculation in the following section. When deriving $H^{\xi_k\text{-TRWA}}(t)$, we neglect the CR terms $-\frac{A}{4}(e^{-i\omega_L t}\sigma_- + e^{i\omega_L t}\sigma_+)$ and the terms of order g_k^2 and higher instead of $H_{\rm CR2}$, which takes into account the CR terms of the dissipative coupling but is not practicable for strong driving since the Rabi-RWA used in this particular treatment is only valid in the weak-driving regime. Finally, we would like to demonstrate that the general treatment of $H^{\zeta\text{-}\xi_k\text{-TRWA}}(t)$ takes into account the CR terms of both the driving and dissipative coupling on the same footing and the omitted $H_2'(t)$ does contribute to physical quantities at $O(g_k^4)$ or $O(g_k^2\frac{A^2}{\omega_L^2}\zeta^2)$. We show the detailed comparison in Sec. IV.

C. Valid parameter regime of our general treatment

In this section, we give a valid parameter space of our general treatment. The main approximation we use is the neglect of $H_2'(t)$ [Eq. (11)] in the transformed Hamiltonian. In a moderately-weak-damping regime, the validity of the neglect of $H_2'(t)$ depends on the effects of higher-frequency driving-related terms ($n\omega_L, n \geqslant 2$), i.e., the fast-oscillating terms that are multiplied by the second- or higher-order Bessel functions. In general, these terms can be safely dropped under certain conditions, as discussed in Ref. [24]. The valid parameter regime of our method depends on both of the ratios A/ω_L and Δ/ω_L. Our method is fully justified when $A/\omega_L < 2$. Especially, for $\Delta/\omega_L \ll 1$, our method works very well even if A/ω_L increases up to 6. Therefore, it is reasonable for our method to study the physics of the driving-parameter space, which is of primary interest to us. For instance, the driving-parameter regime in quantum optics mainly focuses on the resonant driving with the strength $A/\omega_L < 2$. On the other hand, our method is practicable to study the ultrastrong driving cases, such as $\omega_L = 10\Delta$ and $A/\omega_L < 2.5$. All in all, our method can uniformly treat the weak to strong driving, which is beyond the traditional perturbation theory in the amplitude A or the tunneling Δ. Moreover, our treatment is analytically more simple than the FBM approach.

III. MASTER EQUATION AND ITS SOLUTIONS

A. The master equation

Since the effective Hamiltonians in our three treatments have the same mathematical form as $H^{\rm RWA}(t)$, the derivation of master equations for the effective Hamiltonians is similar. In the following, we give the master equation for $\tilde{H}^{\zeta\text{-}\xi_k\text{-TRWA}}$ as an instance. For the sake of simplicity, we omit the superscript ζ-ξ_k-TRWA in the following derivation. We divide $\tilde{H}$ into two parts, $\tilde{H} = \tilde{H}_0 + \tilde{H}_1$, where

$$\tilde{H}_0 = -\frac{1}{2}\left(\tilde{\delta}\sigma_x + \frac{\tilde{A}}{2}\sigma_z\right) + \sum_k (\omega_k - \omega_L) b_k^\dagger b_k \tag{27}$$

is the free Hamiltonian, and

$$\tilde{H}_1 = \frac{1}{2}\sum_k \tilde{g}_k (b_k^\dagger \sigma_- + b_k \sigma_+) \tag{28}$$

is the interaction Hamiltonian. In the interaction picture, the density matrix of the TLS and bath $\tilde{\rho}_{SB}^{\rm I}(t) = e^{i\tilde{H}_0 t}\tilde{\rho}_{SB}(t)e^{-i\tilde{H}_0 t}$ satisfies the equation of motion,

$$\frac{d}{dt}\tilde{\rho}_{SB}^{\rm I}(t) = -i\big[\tilde{H}_1^{\rm I}(t), \tilde{\rho}_{SB}^{\rm I}(t)\big], \tag{29}$$

where superscript I indicates that the operator is in the interaction picture, square brackets [,] is the usual commutator operation, and $\tilde{H}_1^{\rm I}(t) = e^{i\tilde{H}_0 t}\tilde{H}_1 e^{-i\tilde{H}_0 t}$. The differential equation (29) can be integrated formally, yielding the formal solution

$$\tilde{\rho}_{SB}^{\rm I}(t) = \tilde{\rho}_{SB}^{\rm I}(0) - i\int_0^t d\tau \big[\tilde{H}_1^{\rm I}(\tau), \tilde{\rho}_{SB}^{\rm I}(\tau)\big]. \tag{30}$$

Substituting the formal solution (30) into Eq. (29) and taking trace over the bath, we obtain an integrodifferential equation for the reduced density matrix $\tilde{\rho}_S^{\rm I}(t) = {\rm Tr}_B[\tilde{\rho}_{SB}^{\rm I}(t)]$, which reads

$$\frac{d}{dt}\tilde{\rho}_S^{\rm I}(t) = -\int_0^t d\tau {\rm Tr}_B\big[\tilde{H}_1^{\rm I}(t), \big[\tilde{H}_1^{\rm I}(\tau), \tilde{\rho}_{SB}^{\rm I}(\tau)\big]\big]. \tag{31}$$

To proceed, we can invoke Born–Markov approximation [4,23] as usual. It is achieved by replacing $\tilde{\rho}_{SB}^{\rm I}(\tau)$ and $\tilde{H}_1^{\rm I}(\tau)$ with $\tilde{\rho}_S^{\rm I}(t)\rho_B$ and $\tilde{H}_1^{\rm I}(t-\tau)$, respectively, where ρ_B is the density matrix of the bath, and letting the upper limit of the integral go to infinity, and then, we obtain that

$$\frac{d}{dt}\tilde{\rho}_S^{\rm I}(t) = -\int_0^\infty d\tau {\rm Tr}_B\big[\tilde{H}_1^{\rm I}(t), \big[\tilde{H}_1^{\rm I}(t-\tau), \tilde{\rho}_S^{\rm I}(t)\rho_B\big]\big]. \tag{32}$$

After transforming the equation back into the Schrödinger picture, we arrive at the desired master equation at zero temperature,

$$\begin{aligned}\frac{d}{dt}\tilde{\rho}_S(t) = {}& -i[\tilde{H}_{0S}, \tilde{\rho}_S(t)] - \int_0^\infty d\tau \frac{1}{4}\sum_k \tilde{g}_k^2 \\ & \times \big\{e^{-i(\omega_k-\omega_L)\tau}\big[\sigma_+, e^{-i\tilde{H}_{0S}\tau}\sigma_- e^{i\tilde{H}_{0S}\tau}\tilde{\rho}_S(t)\big] \\ & - e^{i(\omega_k-\omega_L)\tau}\big[\sigma_-, \tilde{\rho}_S(t)e^{-i\tilde{H}_{0S}\tau}\sigma_+ e^{i\tilde{H}_{0S}\tau}\big]\big\},\end{aligned} \tag{33}$$

where $\tilde{H}_{0S} = -\frac{1}{2}(\tilde{\delta}\sigma_x + \frac{\tilde{A}}{2}\sigma_z)$ and we have traced out the degrees of freedom of the bath explicitly.

It is convenient to denote the mean value of the μ-component Pauli operator averaged with respect to $\tilde{\rho}_S(t)$ by $\langle\tilde{\sigma}_\mu(t)\rangle = {\rm Tr}_S[\tilde{\rho}_S(t)\sigma_\mu]$ and introduce the real Bloch vector $\langle\vec{\tilde{\sigma}}(t)\rangle = (\langle\tilde{\sigma}_x(t)\rangle, \langle\tilde{\sigma}_y(t)\rangle, \langle\tilde{\sigma}_z(t)\rangle)$. Using these notations, we can easily derive the Bloch equations from the master equation, which can be written in a matrix form,

$$\frac{d}{dt}\langle\vec{\tilde{\sigma}}(t)\rangle = M\langle\vec{\tilde{\sigma}}(t)\rangle + \vec{I}, \tag{34}$$

where the matrix M is given by

$$M = \begin{pmatrix} -\gamma_x & \frac{\tilde{A}}{2} & \gamma_c \\ -\frac{\tilde{A}}{2} & -\gamma_y & \tilde{\delta} \\ 0 & -\tilde{\delta} & -\gamma_z \end{pmatrix}, \tag{35}$$

and $\vec{I} = (\gamma_x, 0, -\gamma_c)$ is the column vector determining the steady-state solutions of the equations. The parameters are defined as

$$\begin{aligned}\gamma_x &= \gamma_y + \gamma_z, \\ \gamma_y &= [\gamma(\omega_L + \tilde{\Omega})\cos^2\phi - \gamma(\omega_L - \tilde{\Omega})\sin^2\phi]\cos(2\phi) \\ &\quad + \gamma(\omega_L)\sin^2(2\phi), \\ \gamma_z &= \gamma(\omega_L + \tilde{\Omega})\cos^2\phi + \gamma(\omega_L - \tilde{\Omega})\sin^2\phi, \\ \gamma_c &= [\gamma(\omega_L)\cos(2\phi) - \gamma(\omega_L + \tilde{\Omega})\cos^2\phi \\ &\quad + \gamma(\omega_L - \tilde{\Omega})\sin^2\phi]\sin(2\phi),\end{aligned} \tag{36}$$

where

$$\cos^2\phi = \frac{1}{2}\left(1 + \frac{\tilde{\delta}}{\tilde{\Omega}}\right), \tag{37}$$

and

$$\tilde{\Omega} = \sqrt{\tilde{\delta}^2 + \tilde{A}^2/4} \tag{38}$$

is the effective Rabi frequency. $\gamma(\omega)$ is the decay rate at frequency ω and its functional form reads

$$\gamma(\omega) = \frac{\pi}{4}\sum_k \tilde{g}_k^2 \delta(\omega-\omega_k) = \pi\tilde{G}(\omega)/4. \tag{39}$$

In general, it is convenient to solve the Bloch equations (34) with the aid of Laplace transform. However, for the steady-state solutions, one just needs to perform following matrix operation:

$$\langle\vec{\tilde{\sigma}}\rangle_{ss} = -M^{-1}\vec{I}, \tag{40}$$

which leads to

$$\langle\tilde{\sigma}_x\rangle_{ss} = \frac{4\gamma_y(\gamma_x\gamma_z-\gamma_c^2)+2\tilde{\delta}(2\tilde{\delta}\gamma_x-\gamma_c\tilde{A})}{2\tilde{\delta}(2\tilde{\delta}\gamma_x-\gamma_c\tilde{A})+\gamma_z(\tilde{A}^2+4\gamma_x\gamma_y)}, \tag{41}$$

$$\langle\tilde{\sigma}_y\rangle_{ss} = \frac{-2\tilde{A}\gamma_x\gamma_z-2\gamma_c(2\tilde{\delta}\gamma_x-\gamma_c\tilde{A})}{2\tilde{\delta}(2\tilde{\delta}\gamma_x-\gamma_c\tilde{A})+\gamma_z(\tilde{A}^2+4\gamma_x\gamma_y)}, \tag{42}$$

$$\langle\tilde{\sigma}_z\rangle_{ss} = \frac{-4\gamma_c\gamma_x\gamma_y+\tilde{A}(2\tilde{\delta}\gamma_x-\gamma_c\tilde{A})}{2\tilde{\delta}(2\tilde{\delta}\gamma_x-\gamma_c\tilde{A})+\gamma_z(\tilde{A}^2+4\gamma_x\gamma_y)}. \tag{43}$$

B. The secular approximation

Since we are interested in the physics in the strong-driving and moderately-weak-damping regimes, it is convenient to treat the problem in the dressed-state picture where one can invoke the secular approximation to simplify the analytical results. Thus, we can rewrite Eqs. (34) in terms of the mean values of the dressed-state operators. The dressed-state operators are related to the bare TLS operators by the relations

$$\begin{aligned} s_x &= \cos(2\phi)\sigma_z - \sin(2\phi)\sigma_x,\\ s_y &= \sigma_y,\\ s_z &= -\cos(2\phi)\sigma_x - \sin(2\phi)\sigma_z. \end{aligned} \tag{44}$$

Denoting the mean values of dressed-state operators by

$$\langle\tilde{s}_\mu(t)\rangle = \mathrm{Tr}[s_\mu\tilde{\rho}_S(t)], \tag{45}$$

and introducing the raising and lowering operators $s_\pm = \frac{1}{2}(s_x \pm i s_y)$ for the dressed states, we can show the following relations:

$$\begin{aligned}\langle\tilde{s}_+(t)\rangle = \langle\tilde{s}_-(t)\rangle^* = \tfrac{1}{2}[&\cos(2\phi)\langle\tilde{\sigma}_z(t)\rangle\\ &-\sin(2\phi)\langle\tilde{\sigma}_x(t)\rangle + i\langle\tilde{\sigma}_y(t)\rangle],\end{aligned} \tag{46}$$

$$\langle\tilde{s}_z(t)\rangle = -\cos(2\phi)\langle\tilde{\sigma}_x(t)\rangle - \sin(2\phi)\langle\tilde{\sigma}_z(t)\rangle. \tag{47}$$

Using Eqs. (46) and (47), we are able to rewrite the Bloch equations in a new form:

$$\frac{d}{dt}\begin{pmatrix}\langle\tilde{s}_+(t)\rangle\\ \langle\tilde{s}_-(t)\rangle\\ \langle\tilde{s}_z(t)\rangle\end{pmatrix} = M'\begin{pmatrix}\langle\tilde{s}_+(t)\rangle\\ \langle\tilde{s}_-(t)\rangle\\ \langle\tilde{s}_z(t)\rangle\end{pmatrix} - \vec{I}', \tag{48}$$

where the elements of matrix M' are given by

$$\begin{aligned} M'_{11} &= M'^*_{22} = i\tilde{\Omega} - \Gamma_{\rm deph},\\ M'_{33} &= -\Gamma_{\rm rel},\\ M'_{12} &= M'_{21} = -\tfrac{1}{4}[\gamma(\omega_L-\tilde{\Omega})+\gamma(\omega_L+\tilde{\Omega})]\sin^2(2\phi),\\ M'_{13} &= M'_{23} = -\tfrac{1}{2}[\gamma(\omega_L+\tilde{\Omega})\cos^2\phi - \gamma(\omega_L-\tilde{\Omega})\sin^2\phi]\\ &\quad\times\sin(2\phi),\\ M'_{31} &= M'_{32} = -\tfrac{1}{2}\gamma(\omega_L)\sin(4\phi). \end{aligned} \tag{49}$$

Here,

$$\begin{aligned}\Gamma_{\rm deph} = {}&\gamma(\omega_L+\tilde{\Omega})\cos^4\phi + \gamma(\omega_L-\tilde{\Omega})\sin^4\phi\\ &+\gamma(\omega_L)\sin^2(2\phi),\end{aligned} \tag{50}$$

$$\Gamma_{\rm rel} = 2\gamma(\omega_L+\tilde{\Omega})\cos^4\phi + 2\gamma(\omega_L-\tilde{\Omega})\sin^4\phi, \tag{51}$$

are the dephasing and relaxation rate, respectively. The inhomogeneous column vector is $\vec{I}' = (\Gamma_c,\Gamma_c,\Gamma_p)$ with

$$\begin{aligned}\Gamma_c = \tfrac{1}{2}[&\gamma(\omega_L)+\gamma(\omega_L+\tilde{\Omega})\cos^2\phi\\ &+\gamma(\omega_L-\tilde{\Omega})\sin^2\phi]\sin(2\phi),\end{aligned} \tag{52}$$

$$\Gamma_p = 2\gamma(\omega_L+\tilde{\Omega})\cos^4\phi - 2\gamma(\omega_L-\tilde{\Omega})\sin^4\phi. \tag{53}$$

Up until now, we just change the representation without any approximation. To proceed the treatment analytically, we invoke the secular approximation; namely, the neglect of the nondiagonal elements of M' and Γ_c of the inhomogeneous part $\vec{I}'$, which is justified when the driving is strong enough or largely detuned $[\tilde{\Omega} \gg \gamma(\omega_L),\gamma(\omega_L\pm\tilde{\Omega})]$. By the secular approximation, Eqs. (48) gives the decoupled equations

$$\frac{d}{dt}\langle\tilde{s}_+(t)\rangle = (-\Gamma_{\rm deph}+i\tilde{\Omega})\langle\tilde{s}_+(t)\rangle, \tag{54}$$

$$\frac{d}{dt}\langle\tilde{s}_z(t)\rangle = -\Gamma_{\rm rel}\langle\tilde{s}_z(t)\rangle - \Gamma_p, \tag{55}$$

which lead to the solutions of the simple form

$$\langle\tilde{s}_+(t)\rangle = \langle\tilde{s}_-(t)\rangle^* = -\tfrac{1}{2}\cos(2\phi)e^{(-\Gamma_{\rm deph}+i\tilde{\Omega})t}, \tag{56}$$

$$\langle\tilde{s}_z(t)\rangle = -\tfrac{\Gamma_p}{\Gamma_{\rm rel}}(1-e^{-\Gamma_{\rm rel}t}) + \sin(2\phi)e^{-\Gamma_{\rm rel}t}, \tag{57}$$

where we have used the initial condition $\langle\sigma_z(0)\rangle = -1$.

By using the inverse relations of Eqs. (44), we can derive the explicit form of $\langle\tilde{\sigma}_\mu(t)\rangle$, which reads

$$\begin{aligned}\langle\tilde{\sigma}_x(t)\rangle = {}&\cos(\tilde{\Omega}t)e^{-\Gamma_{\rm deph}t}\sin(2\phi)\cos(2\phi)\\ &+\left[\frac{\Gamma_p}{\Gamma_{\rm rel}}(1-e^{-\Gamma_{\rm rel}t}) - \sin(2\phi)e^{-\Gamma_{\rm rel}t}\right]\cos(2\phi),\end{aligned} \tag{58}$$

$$\langle\tilde{\sigma}_y(t)\rangle = -\sin(\tilde{\Omega}t)e^{-\Gamma_{\rm deph}t}\cos(2\phi), \tag{59}$$

$$\begin{aligned}\langle\tilde{\sigma}_z(t)\rangle = {}&-\cos(\tilde{\Omega}t)e^{-\Gamma_{\rm deph}t}\cos^2(2\phi)\\ &+\left[\frac{\Gamma_p}{\Gamma_{\rm rel}}(1-e^{-\Gamma_{\rm rel}t}) - e^{-\Gamma_{\rm rel}t}\sin(2\phi)\right]\sin(2\phi).\end{aligned} \tag{60}$$

Similarly, the master equations and solutions for the other three Hamiltonians can be obtained by the same procedure for $H^{\zeta\text{-}\xi_k\text{-TRWA}}(t)$. Alternatively, one can directly obtain the corresponding results for the other Hamiltonians from those for $H^{\zeta\text{-}\xi_k\text{-TRWA}}(t)$ by replacing the quantities of the same position in the effective Hamiltonian. For instance, when the parameters in above equations are replaced as follows:

$$\tilde{A} \to A, \quad \tilde{\delta} \to \delta_0, \quad \tilde{G}(\omega) \to G(\omega), \tag{61}$$

we obtain the results for the traditional RWA approach.

IV. DYNAMICS OF POPULATION DIFFERENCE AND COHERENCE

In this section, we start to identify the relation between our method and FBM approach and demonstrate the physical effects induced by the CR terms by comparing the results of the three RWA-like and RWA methods. In the following, because we consider the weak-dissipative-coupling case, the renormalization of the tunneling can be well approximated as $\eta \approx 1$ for $H^{\zeta\text{-}\xi_k\text{-TRWA}}(t)$ and $H^{\xi_k\text{-TRWA}}(t)$. Therefore, in the weak-driving limit, the resonance condition $\omega_L = \eta\Delta$ for $H^{\zeta\text{-}\xi_k\text{-TRWA}}(t)$ and $H^{\xi_k\text{-TRWA}}(t)$ is nearly the same as the resonant condition $\omega_L = \Delta$ for $H^{\text{RWA}}(t)$.

Henceforth, for the sake of simplicity, we use the different superscript of the effective Hamiltonian to denote the four methods of the RWA form and their results, e.g., the ζ-ξ_k-TRWA represents the results calculated by exactly solving the master equation for $H^{\zeta\text{-}\xi_k\text{-TRWA}}(t)$ while the ζ-ξ_k-TRWA + SA stands for the analytical results calculated within the secular approximation (SA) for $H^{\zeta\text{-}\xi_k\text{-TRWA}}(t)$.

We now introduce how to calculate the physical quantities of primary interest within our formalism. In general, it is convenient to calculate the physical quantities by using the reduced density matrix governed by the effective Hamiltonian, which is obtained by $\tilde{\rho}_S(t) = \text{Tr}_B[\tilde{\rho}_{SB}(t)]$. Notice that $\tilde{\rho}_{SB}(t)$ is related to the density matrix of the total system $\rho_{SB}(t)$ in the original frame by $\tilde{\rho}_{SB}(t) = R(t)e^{S(t)}\rho_{SB}(t)e^{-S(t)}R^\dagger(t)$. Thus, we can give the population difference for the ζ-ξ_k-TRWA as follows:

$$\begin{aligned}
\langle\sigma_z(t)\rangle &= \text{Tr}_{SB}[\sigma_z\rho_{SB}(t)] \\
&= \text{Tr}_{SB}[R(t)e^{S(t)}\sigma_z e^{-S(t)}R^\dagger(t)\tilde{\rho}_{SB}(t)] \\
&= \text{Tr}_{SB}[R(t)\sigma_z R^\dagger(t)\tilde{\rho}_{SB}(t)] \\
&= \text{Tr}_S[(\sigma_+ e^{i\omega_L t} + \sigma_- e^{-i\omega_L t})\tilde{\rho}_S(t)] \\
&= \langle\tilde{\sigma}_z(t)\rangle\cos(\omega_L t) - \langle\tilde{\sigma}_y(t)\rangle\sin(\omega_L t).
\end{aligned} \tag{62}$$

When the Bloch equations (34) are exactly solved, the population difference can be completely determined. In addition, we can give simple analytical solutions by using the approximate results for $\langle\tilde{\sigma}_\mu(t)\rangle$ given in Eqs. (58)–(60), which lead to the analytical expression

$$\begin{aligned}
\langle\sigma_z(t)\rangle = &\cos(\omega_L t)e^{-\Gamma_{\text{rel}}t}\left[\frac{\Gamma_p}{\Gamma_{\text{rel}}}(e^{\Gamma_{\text{rel}}t}-1) - \sin(2\phi)\right]\sin(2\phi) \\
&- \cos(\omega_L t)\cos(\tilde{\Omega}t)e^{-\Gamma_{\text{deph}}t}\cos^2(2\phi) \\
&+ \sin(\omega_L t)\sin(\tilde{\Omega}t)e^{-\Gamma_{\text{deph}}t}\cos(2\phi).
\end{aligned} \tag{63}$$

Note that the approximate solutions are derived with the aid of the secular approximation in the dressed-state picture, which means that the solutions are valid for a sufficiently strong driving or a largely detuned driving. In addition, when replacing the involved physical quantities ($\tilde{A}$, $\tilde{\Delta}$, $\tilde{\Omega}$, and $\tilde{g}_k$) of the general treatment by the corresponding bare or modified quantities of the other three effective Hamiltonians (H^{RWA}, $H^{\xi_k\text{-TRWA}}$, and $H^{\zeta\text{-TRWA}}$), respectively, one immediately obtain the corresponding population difference with the same mathematical forms as the formulas (62) and (63) for the ζ-ξ_k-TRWA method.

Similarly, the coherence for the ζ-ξ_k-TRWA method can be evaluated as the population difference,

$$\begin{aligned}
\langle\sigma_x(t)\rangle &= \text{Tr}_{SB}[\sigma_x\rho_{SB}(t)] \\
&= \text{Tr}_{SB}[R(t)e^{S(t)}\sigma_x e^{-S(t)}R^\dagger(t)\tilde{\rho}_{SB}(t)] \\
&= \eta\{\cos X_2\langle\tilde{\sigma}_x(t)\rangle + \sin X_2[\langle\tilde{\sigma}_z(t)\rangle\sin(\omega_L t) \\
&\quad + \langle\tilde{\sigma}_y(t)\rangle\cos(\omega_L t)]\},
\end{aligned} \tag{64}$$

where $X_2 = \frac{A}{\omega_L}\zeta\sin(\omega_L t)$. The formula shows that the CR terms of the driving lead to a multifrequency dependence of the coherence because of the time factors $\cos X_2$ and $\sin X_2$. However, within the Rabi-RWA, $\cos X_2$ and $\sin X_2$ are replaced with 1 and 0, respectively, and the coherence is simply determined by the mean value of σ_x with respect to the reduced density matrix for the Hamiltonian H^{RWA}.

Before discussing the strongly driven dynamics, we show the relations between the RWA method and the other three RWA-like methods in the weak-driving and weak-damping limit. It is well known that the RWA is reasonably used to describe the physics of interest when both the driving and damping are weak enough. In the following we demonstrate that our methods agree with the RWA one in the weak-driving and weak-damping limit by the asymptotic behavior of the modified quantities of the ζ-ξ_k-TRWA. In the weak-damping limit, $\eta = 1$. On the other hand, in the weak-driving limit $A/\omega_L \ll 1$, we expand $J_1(\frac{A}{\omega_L}\zeta)$ as $J_1(\frac{A}{\omega_L}\zeta) \approx \frac{A}{2\omega_L}\zeta + O(\frac{A^3\zeta^3}{\omega_L^3})$, and substituting it into Eq. (13) we obtain $\zeta = \frac{1}{2}$, and $\tilde{A} = A$. Moreover, we have $J_0(\frac{A}{\omega_L}\zeta) \approx 1$ and then, $\tilde{\delta} \approx \Delta - \omega_L$. Therefore, the modified physical quantities recover the bare ones of the RWA. Besides, all decay rates of the ζ-ξ_k-TRWA (39) at the three transition frequencies ω_L and $\omega_L \pm \tilde{\Omega}$ tend to the same value of $\pi G(\Delta)/4$ when $\omega_L = \Delta$ and $A/\omega_L \ll 1$. Thus, it turns out that the RWA result is the limit of the three methods (the ζ-TRWA, ξ_k-TRWA and ζ-ξ_k-TRWA methods) in the weak-driving and weak-damping limit. In the following, we study the strongly driven dynamics by comparing the results of these methods and the FBM approach.

A. Comparison of our method and Floquet–Born–Markov approach

In this section, we reveal the relation between the ζ-TRWA method and the FBM approach (whose results will be denoted by FBM in plots). The Floquet Hamiltonian and the corresponding master equation used for evaluating the dynamics in this work are referred to as Eqs. (4) and (38) in Ref. [14], where the general time-dependent Bloch–Redfield tensor in the master equation (38) is replaced by Eq. (47) in Ref. [14] (this is the so-called moderate rotating-wave approximation—MRWA).

We first compare the population difference predicted by the ζ-TRWA, ζ-TRWA + SA, and FBM approaches. We plot the population difference as a function of time t for the three methods with $A/\omega_L = 0.1$ in Fig. 1 and $A/\omega_L = 1$ in Fig. 2.

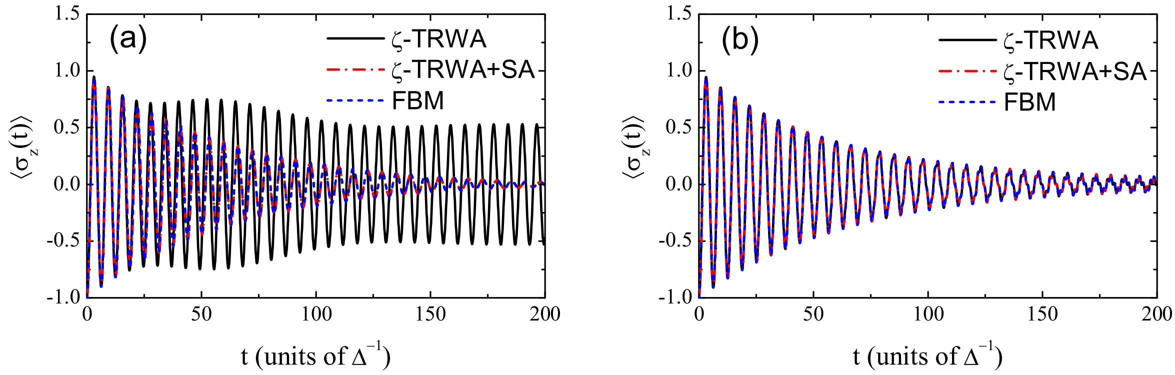

FIG. 1. (Color online) $\langle\sigma_z(t)\rangle$ as a function of t is shown for $A/\omega_L = 0.1$, $\alpha = 0.01$, and (a) $\omega_L = \Delta$ and (b) $\omega_L = 4\Delta$.

We find that, when $A/\omega_L = 0.1$ or 1, the ζ-TRWA + SA and FBM approaches give almost the same results. In fact, we verify numerically that, in the region of $A/\omega_L < 2$, the ζ-TRWA + SA approach gives almost the same dissipative dynamics ($\alpha \neq 0$) and nondissipative dynamics ($\alpha = 0$) as that of the FBM approach. It means that, in some senses, the approximation used in ζ-TRWA + SA is equivalent to those used in the FBM approach. In other words, the role of MRWA of treating the dissipation in the FBM approach is approximately equivalent to that of the two approximations in our treatment: (i) the neglect of $\frac{1}{2}\sum_k g_k(b_k\sigma_- + b_k^\dagger\sigma_+)$ in the transformed Hamiltonian; (ii) the secular approximation. Although the results of the ζ-TRWA + SA agrees quite well with those of the FBM approach in the certain parameter region, they may not coincide with those of the ζ-TRWA because the secular approximation used can be invalid for moderately weak driving, such as the case in Fig. 1(a).

In Figs. 3(a)–3(d), we plot the population difference and coherence for the ratio $A/\omega_L = 2.5$, which is beyond the region $A/\omega_L < 2$. It is found that ζ-TRWA and ζ-TRWA + SA give the same results and their time evolution differs from that of the FBM approach in particular when $\omega_L = \Delta$ even though the oscillatory frequency of their stable states is the same. The difference can be understood by recalling the approximation invoked in $\tilde{H}^{\zeta\text{-TRWA}}$. When deriving $\tilde{H}^{\zeta\text{-TRWA}}$, we neglect the second- and higher-order Bessel functions of the first kind, whose influence depends on the ratios A/ω_L and Δ/ω_L. This approximation can be justified in the region of $A/\omega_L < 2$. Therefore, we see the excellent agreement of the results given by the ζ-TRWA + SA and FBM approaches. However, when A/ω_L is out of the region, the approximation of $\tilde{H}^{\zeta\text{-TRWA}}$ might not be well justified and it arises that the difference between our method and FBM approach. Nevertheless, Figs. 3(c) and 3(d) show that, when $A/\omega_L = 2.5$ and $\omega \gg \Delta$, the ζ-TRWA can also provide a good description compared with the FBM approach.

To end this section, we give some remarks on the comparison of the ζ-TRWA and FBM approaches. First, from the above comparisons, we verify the valid driving-parameter space of our method given in Sec. II C and clarify the relation between the ζ-TRWA and FBM approaches. It is obvious that our method is analytically more simple and numerically more efficient than the FBM approach. Second, in some senses, the MRWA's role invoked in the FBM approach is equivalent to the two specific approximations in the ζ-TRWA treatment: (i) the neglect of the CR terms $\frac{1}{2}\sum_k g_k(b_k^\dagger\sigma_+ + b_k\sigma_-)$ in the transformed Hamiltonian and (ii) the secular approximation. We should point out that, although the secular approximation is generally valid in the strong driving regimes, the neglect of the CR terms $\frac{1}{2}\sum_k g_k(b_k^\dagger\sigma_+ + b_k\sigma_-)$ may not be justified since the CR terms of the dissipative coupling lead to the renormalization of the spectral density [see Eq. (26)]. In the following, we discuss how the effects of the renormalized spectrum modify significantly the time evolution under certain conditions.

B. Reduction of dephasing and relaxation rates

In this section, we examine the role of the CR terms of the driving and dissipative coupling in the interference between driving and dissipation. To begin with, we compare the time

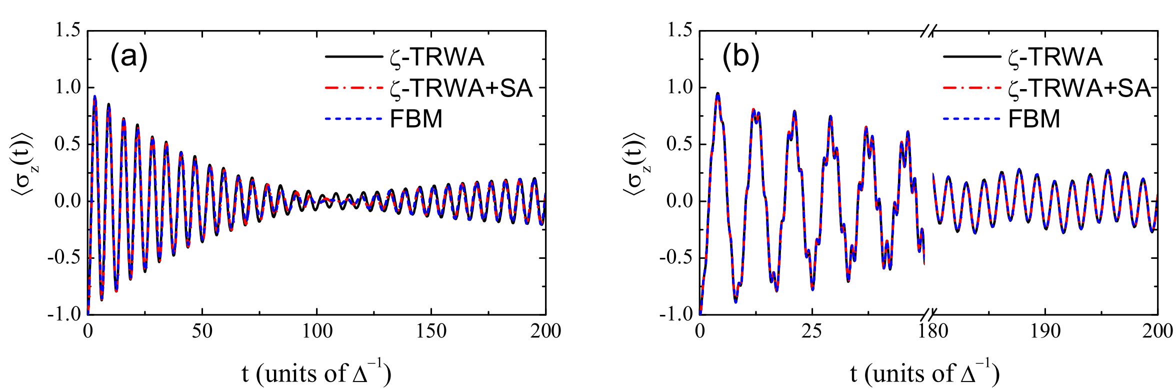

FIG. 2. (Color online) $\langle\sigma_z(t)\rangle$ as a function of t is shown for $A/\omega_L = 1$, $\alpha = 0.01$, and (a) $\omega_L = \Delta$ and (b) $\omega_L = 4\Delta$.

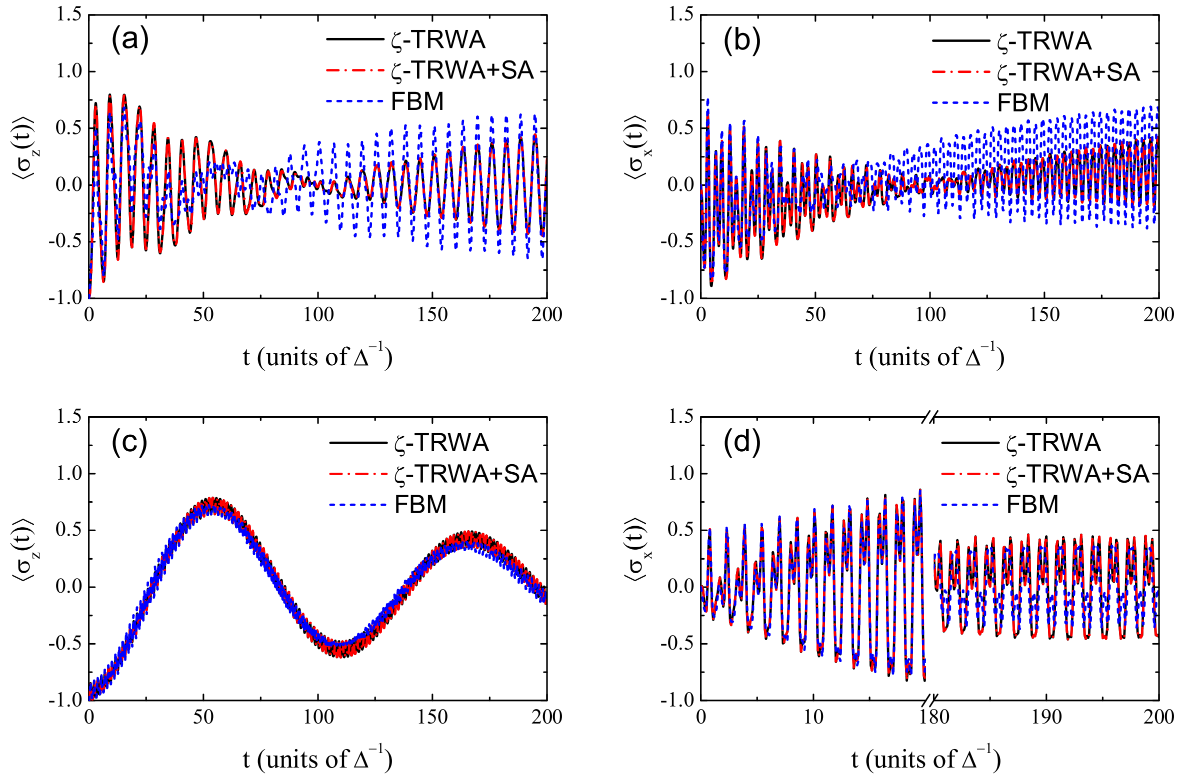

FIG. 3. (Color online) [(a), (c)] $\langle\sigma_z(t)\rangle$ and [(b), (d)] $\langle\sigma_x(t)\rangle$ as functions of t are shown for $A/\omega_L = 2.5$, $\alpha = 0.01$, and driving frequencies [(a), (b)] $\omega_L = \Delta$ and [(c), (d)] $\omega_L = 4\Delta$.

evolution of the population difference and coherence, which is shown in Figs. 4(a)–4(d) and obtained by the ζ-TRWA, ξ_k-TRWA, and ζ-ξ_k-TRWA for $A/\omega_L = 1$. It is obvious that the results of the ξ_k-TRWA method differ from those of the other two methods for both on-resonance driving and off-resonance driving, which results from the break-down of the Rabi-RWA used in the ξ_k-TRWA approach. On the other hand, when comparing the results of the ζ-TRWA and ζ-ξ_k-TRWA approaches, one finds that those of the two treatments coincide for the off-resonance case but differ from each other for the on-resonance case. It indicates that the CR terms $\frac{1}{2}\sum_k g_k(b_k^\dagger\sigma_+ + b_k\sigma_-)$ omitted in ζ-TRWA indeed have significant effects on the dynamics because the driving is strong enough for the on-resonance case. To understand the phenomenon, we use the analytical results obtained by the secular approximation (Sec. III B) to analyze the relaxation and dephasing processes.

In order to demonstrate the difference of the time evolution, we check the behaviors of the dephasing rate [Eq. (50)] and the relaxation rate [Eq. (51)], which characterize the dephasing and relaxation processes of the dressed states. We show the comparison of the relaxation rates in Fig. 5(a) and dephasing rates in Fig. 5(b) given by the four methods (the RWA, ζ-TRWA, ξ_k-TRWA, and ζ-ξ_k-TRWA methods) for $\omega_L = \Delta$. In comparison with the RWA results, the three TRWA methods predict that both kinds of rates decrease as A increases. Besides, one finds that the ζ-ξ_k-TRWA method predicts the smallest both kinds of rates than the other three methods. In the following, we take the relaxation rate as an instance to show how the CR terms modify its value (the behavior of the dephasing rate can be understood similarly).

First, we show the effects of the driving CR terms on the relaxation rate of the ζ-TRWA method as compared with those of the RWA method since the ζ-ξ_k-TRWA method has the same treatment of the driving CR terms as the ζ-TRWA method. When $\omega_L = \Delta$, the relaxation rate has the form

$$\Gamma_{\rm rel}^{\zeta\text{-TRWA}} \approx \frac{\pi}{4}G(\Delta) + \pi\alpha\tilde{\delta} + O(\tilde{\delta}^2). \qquad (65)$$

However, the RWA method gives a constant decay rate $\frac{\pi}{4}G(\Delta)$ since $\delta_0 = 0$. As the effective detuning $\tilde{\delta} = J_0(\frac{A}{\omega_L}\zeta)\Delta - \Delta < 0$ decreases with increasing A, one can see that the relaxation rate decreases accordingly. Physically, the nonzero effective detuning is related to the Bloch–Siegert-type correction [28]. As is shown in Ref. [24], by the effective Rabi frequency $(\tilde{\delta}^2 + \tilde{A}^2/4)^{1/2}$, we have calculated the Bloch–Siegert shift up to fourth order in A, which is consistent with Floquet theory [21]. In fact, the Bloch–Siegert shift has been embodied in both the effective detuning $\tilde{\delta}$ and the modified driving strength $\tilde{A}$ in our formalism.

Second, we show the role of the dissipative CR terms of the dissipative interaction of the ξ_k-TRWA as compared to the RWA results since the ζ-ξ_k-TRWA method has the same treatment of the dissipative CR terms as the ξ_k-TRWA method. In our formalism, the dissipative CR terms lead to the renormalized spectral density given by Eq. (26). For $A \leqslant \Delta$ and $\omega_L = \eta\Delta$, the relaxation rate for the ξ_k-TRWA is given

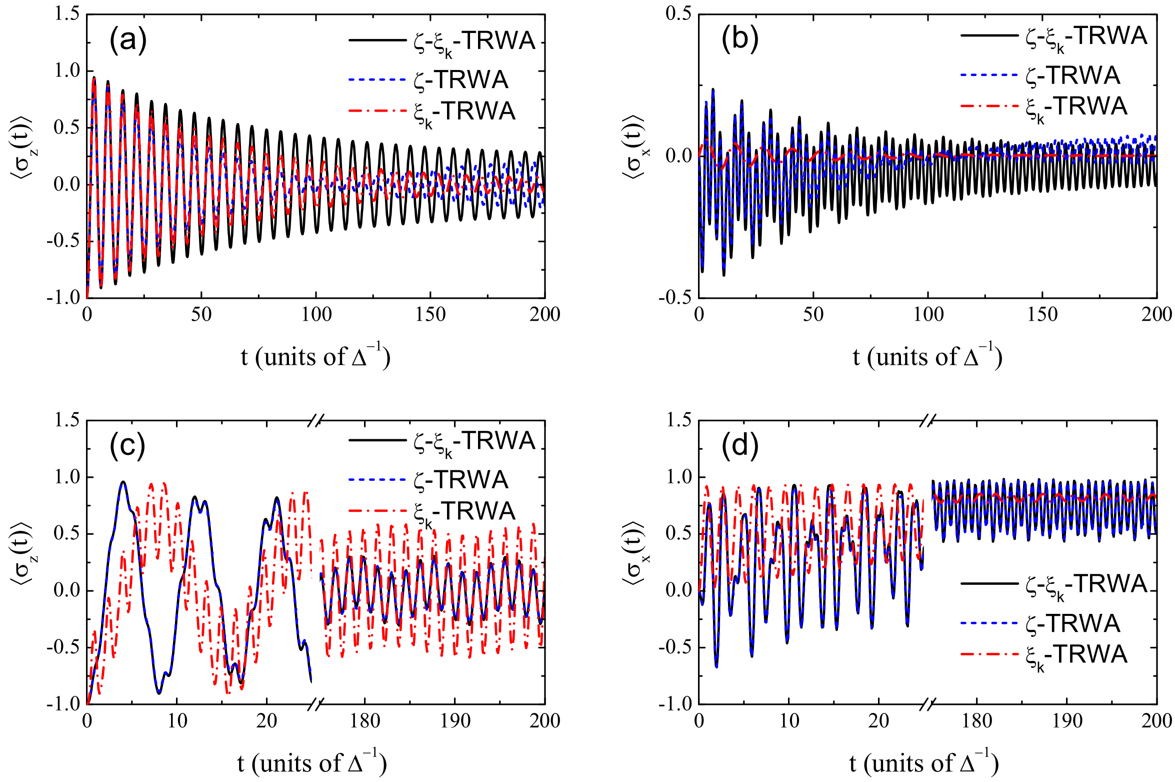

FIG. 4. (Color online) [(a), (c)] $\langle\sigma_z(t)\rangle$ and [(b), (d)] $\langle\sigma_x(t)\rangle$ as functions of t are shown for $A/\omega_L = 1$, $\alpha = 0.01$, and driving frequencies [(a), (b)] $\omega_L = \Delta$ and [(c), (d)] $\omega_L = 4\Delta$.

as [22]

$$\Gamma_{\rm rel}^{\xi_k\text{-TRWA}} = \frac{\pi}{4}G(\eta\Delta)\left[1-\frac{3}{16}\left(\frac{A}{\eta\Delta}\right)^2\right] \times\left[1-\frac{1}{16}\left(\frac{A}{\eta\Delta}\right)^2\right]^{-2}. \quad (66)$$

It shows that the rate reduces as A increases, which results from the modified factor $(2\eta\Delta)^2/(\omega+\eta\Delta)^2$ of the spectral density $\tilde{G}(\omega)$. Furthermore, the modified factor arises from the parameter ξ_k of the generator with the form $\xi_k = \omega_k/(\omega_k + \eta\Delta)$, which depends on the boson frequency [26]. Physically, $\xi_k \approx 1$ when $\omega_k \gg \eta\Delta$, which means that the bath modes can follow the motion of the system. When $\omega_k \ll \eta\Delta$, ξ_k becomes very small, which means that the corresponding bath modes are too slow to follow the motion. However, the RWA method does not capture this nature of the bath modes.

Last not the least, we show the feature of the interference between driving and dissipation in the presence of two types of the CR terms. It is obvious that the ζ-ξ_k-TRWA takes into account the effects of two types of CR terms, which can be represented by the renormalized physical quantities $\tilde{\delta}$, $\tilde{A}$, $\tilde{g}_k$, etc. Moreover, the effects of the renormalized coupling $\tilde{g}_k$ can be described by $\tilde{G}(\omega)$, which possesses the factor

$$F = \left[\frac{2J_0\left(\frac{A}{\omega_L}\zeta\right)\eta\Delta}{\omega + J_0\left(\frac{A}{\omega_L}\zeta\right)\eta\Delta}\right]^2.$$

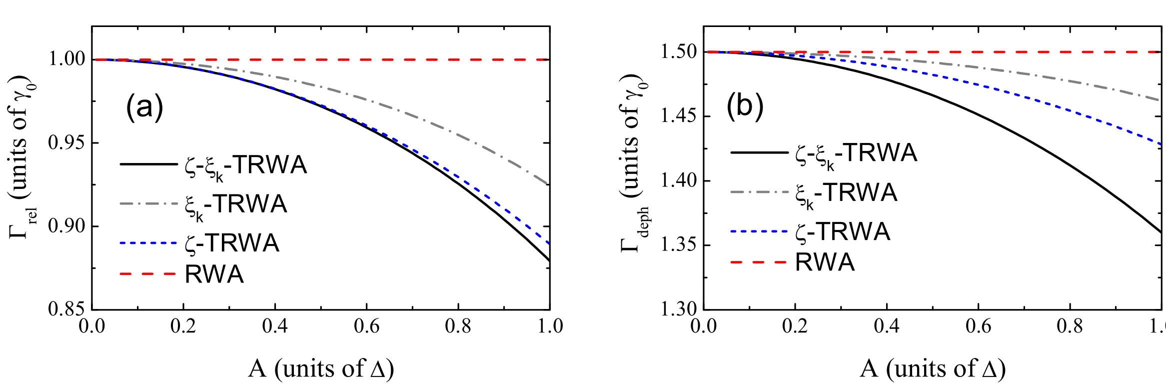

FIG. 5. (Color online) (a) The relaxation rate and (b) the dephasing rate as functions of A are shown for $\alpha = 0.01$ and $\omega_L = \Delta$. $\gamma_0 = \pi G(\Delta)/4$ is set as units.

The factor is related to the renormalized tunneling $J_0(\frac{A}{\omega_L}\zeta)\eta\Delta$, in which the renormalized factor $J_0(\frac{A}{\omega_L}\zeta)$ comes from the driving CR interactions and the other factor η results from the dissipation-CR couplings. In addition, when $\omega \to J_0(\frac{A}{\omega_L}\zeta)\eta\Delta$, F tends to 1, and then $\tilde{G}(\omega) \to G(\omega)$. In other words, in some senses, the modification of the spectral density becomes unobservable. However, when $\omega \neq J_0(\frac{A}{\omega_L}\zeta)\eta\Delta$, $\tilde{G}(\omega)$ is different from $G(\omega)$, which contributes to the distinct dynamic characters of the ζ-ξ_k-TRWA treatment in comparison with the time evolution of the ζ-TRWA [see Figs. 4(a) and 4(b)].

From the above discussion, we conclude that the CR terms lead to the corrections of physical quantities and the renormalization of the spectral density, which contribute important influence to the relaxation and dephasing processes. Moreover, in strong-driving regimes and for the on-resonance driving $\omega_L = \Delta$, the effects of the driving CR and dissipative CR terms cause the reduction in the the relaxation and dephasing rates. For the properly off-resonance driving, the renormalization of the spectral density becomes unobservable and the effect of the CR terms of the driving dominates the difference between RWA and non-RWA results.

C. Effects of counter-rotating terms on steady-state properties

We now discuss the effects of the CR terms on the properties of the steady state of the TLS, i.e., $\langle\sigma_z(t\to\infty)\rangle$ and $\langle\sigma_x(t\to\infty)\rangle$. We first consider the behavior of $\langle\sigma_z(t)\rangle$ in the long-time limit, which is a periodic oscillation with the driving frequency ω_L,

$$\begin{aligned}\langle\sigma_z(t\to\infty)\rangle &= \langle\tilde{\sigma}_z\rangle_{ss}\cos(\omega_L t) - \langle\tilde{\sigma}_y\rangle_{ss}\sin(\omega_L t)\\ &= P_\infty\cos(\omega_L t+\varphi), \end{aligned} \tag{67}$$

where

$$P_\infty = \sqrt{\langle\tilde{\sigma}_z\rangle_{ss}^2 + \langle\tilde{\sigma}_y\rangle_{ss}^2} \tag{68}$$

is the steady oscillation amplitude and the phase φ satisfies $\tan\varphi = \langle\tilde{\sigma}_y\rangle_{ss}/\langle\tilde{\sigma}_z\rangle_{ss}$. The formula (67) is derived without the secular approximation. In Fig. 6(a), we show the amplitude P_∞ given by the four methods as a function of A under the resonance condition $\omega_L = \Delta$. We note that, in the weak-driving limit, the oscillation amplitudes given by the four methods agree with each other. However, they are different in the strong-driving limit, which indicates that the CR terms can significantly modify the steady state. The influence of the CR terms on P_∞ can be revealed by the similar analysis as the former subsection. In the strong-driving case, P_∞ is determined by the behavior of $\langle\tilde{\sigma}_z\rangle_{ss}$ since $\langle\tilde{\sigma}_y\rangle_{ss}$ becomes very small when the driving amplitude A is large enough. For $\omega_L = \Delta$, it is easy to derive explicit forms of $\langle\tilde{\sigma}_z\rangle_{ss}$ for the three methods,

$$\langle\tilde{\sigma}_z\rangle_{ss}^{\text{RWA}} = \frac{A}{2\Delta}, \tag{69}$$

$$\langle\tilde{\sigma}_z\rangle_{ss}^{\zeta\text{-TRWA}} \approx \frac{\tilde{A}}{2\Delta} + \left(\frac{4\Delta^2-\tilde{A}^2}{\tilde{A}\Delta^2}\right)\tilde{\delta}, \tag{70}$$

$$\langle\tilde{\sigma}_z\rangle_{ss}^{\xi_k\text{-TRWA}} = \frac{A^3}{2\Delta(16\Delta^2-3A^2)}. \tag{71}$$

It is the driving CR terms that induce the corrections to $\langle\tilde{\sigma}_z\rangle_{ss}^{\zeta\text{-TRWA}}$ and lead to the difference between $\langle\tilde{\sigma}_z\rangle_{ss}^{\zeta\text{-TRWA}}$ and $\langle\tilde{\sigma}_z\rangle_{ss}^{\text{RWA}}$, while the difference between $\langle\tilde{\sigma}_z\rangle_{ss}^{\xi_k\text{-TRWA}}$ and $\langle\tilde{\sigma}_z\rangle_{ss}^{\text{RWA}}$ results from the renormalization of the spectral density included in $\langle\tilde{\sigma}_z\rangle_{ss}^{\xi_k\text{-TRWA}}$. $\langle\tilde{\sigma}_z\rangle_{ss}^{\zeta\text{-}\xi_k\text{-TRWA}}$, which is different from those of the other three methods, includes the effects of both the driving and dissipative CR terms. In Fig. 6(b), we show $\langle\tilde{\sigma}_z\rangle_{ss}$ of the four methods for comparison. It is interesting to find that either the driving CR terms or the dissipative CR terms reduce the values of $\langle\tilde{\sigma}_z\rangle_{ss}$. Especially, when the two types of CR terms are taken into account, ζ-ξ_k-TRWA predicts negative values of $\langle\tilde{\sigma}_z\rangle_{ss}$ in strong contrast to the positive values given by the other three methods.

We show how the CR terms modify P_∞ upon variation of ω_L for a fixed A. Figure 7(a) shows P_∞ as a function of ω_L for a finite driving $A = 0.5\Delta$. It is notable that when $\omega_L < 0.75\Delta$ or $\omega_L > 1.25\Delta$, the ζ-ξ_k-TRWA result is in good agreement with the ζ-TRWA result, and the ξ_k-TRWA result is consistent with the RWA one. This indicates that, for the large-detuning case, the renormalized effects of the spectral density in the ζ-ξ_k-TRWA (ξ_k-TRWA) treatment becomes unobservable by comparison with the ζ-TRWA (RWA) results. However, the effect of the driving CR terms gives rise to the difference between the ζ-TRWA (ζ-ξ_k-TRWA) and RWA (ξ_k-TRWA) results. The phenomenon can be understood analytically. For $|\omega_L - \Delta| \gg 0$ and moderate strong-driving case, we have

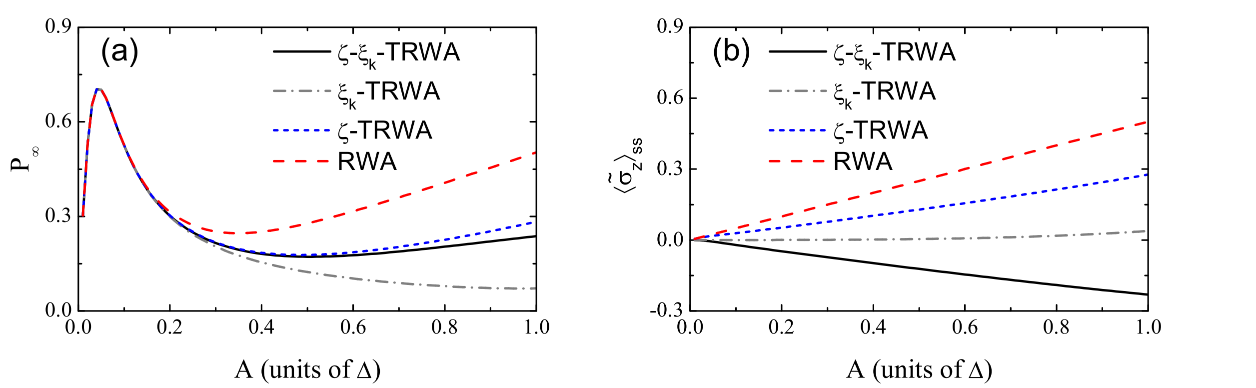

FIG. 6. (Color online) (a) The steady oscillation amplitude P_∞ and (b) $\langle\tilde{\sigma}_z\rangle_{ss}$ as functions of A are shown for $\alpha = 0.01$ and driving frequency $\omega_L = \Delta$.

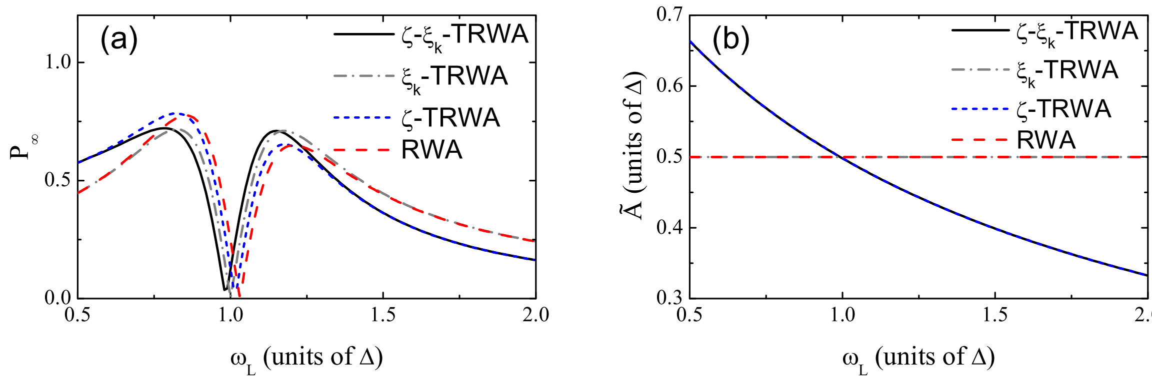

FIG. 7. (Color online) (a) The steady oscillation amplitude P_∞ and (b) the modified driving strength $\tilde{A}$ as functions of ω_L are shown for $A = 0.5\Delta$ and $\alpha = 0.01$.

$\Gamma_p/\Gamma_{\rm rel} \simeq 1$, which is independent of the spectral density. Therefore, $\langle\tilde{\sigma}_z\rangle_{ss}$ takes the following simple form:

$$\langle\tilde{\sigma}_z\rangle_{ss}^{\zeta\text{-}\xi_k\text{-TRWA}} = \langle\tilde{\sigma}_z\rangle_{ss}^{\zeta\text{-TRWA}} = \frac{\tilde{A}}{2\sqrt{\tilde{\delta}^2 + \tilde{A}^2/4}}, \quad (72)$$

$$\langle\tilde{\sigma}_z\rangle_{ss}^{\xi_k\text{-TRWA}} = \langle\tilde{\sigma}_z\rangle_{ss}^{\text{RWA}} = \frac{A}{2\sqrt{\delta_0^2 + A^2/4}}. \quad (73)$$

Since the modified quantities ($\tilde{A}$ and $\tilde{\delta}$) induced by the driving CR terms are different from the bare ones (A and Δ), they determine the difference between the two types of stable states. We show $\tilde{A}$ as a function of ω_L in Fig. 7(b), which indicates that this difference can be enhanced for large detuning.

We discuss the steady-state behavior of the coherence in the following. For the ζ-ξ_k-TRWA method (the ζ-TRWA case is similar), we have

$$\langle\sigma_x(t\to\infty)\rangle = \langle\sigma_x\rangle_{\rm sv} + \sum_{n=1}^{\infty} Q_{n,\infty}\cos(2n\omega_L t - \varphi_n), \quad (74)$$

where

$$\langle\sigma_x\rangle_{\rm sv} = \eta J_0\left(\frac{A}{\omega_L}\zeta\right)\langle\tilde{\sigma}_x\rangle_{ss} + \eta J_1\left(\frac{A}{\omega_L}\zeta\right)\langle\tilde{\sigma}_z\rangle_{ss} \quad (75)$$

is the static value of the coherence,

$$Q_{n,\infty} = \eta\sqrt{N_n^2 + M_n^2} \quad (76)$$

is the amplitude of the oscillation with frequency $2n\omega_L$, and the phase is given by $\varphi_n = \arctan(M_n/N_n)$. The parameters N_n and M_n read

$$N_n = 2J_{2n}\left(\frac{A}{\omega_L}\zeta\right)\langle\tilde{\sigma}_x\rangle_{ss} + \left[J_{2n+1}\left(\frac{A}{\omega_L}\zeta\right) - J_{2n-1}\left(\frac{A}{\omega_L}\zeta\right)\right]\langle\tilde{\sigma}_z\rangle_{ss}, \quad (77)$$

$$M_n = \left[J_{2n+1}\left(\frac{A}{\omega_L}\zeta\right) + J_{2n-1}\left(\frac{A}{\omega_L}\zeta\right)\right]\langle\tilde{\sigma}_y\rangle_{ss}. \quad (78)$$

We note that, in the long-time limit, the coherence oscillates around its static value with even multiples of driving frequency $2n\omega_L$. However, when the driving CR terms are neglected, the coherence remains constant after a sufficiently long time. Notice that the static value of the coherence is mainly determined by $\langle\tilde{\sigma}_x\rangle_{ss}$. In the strong-driving limit, $\langle\tilde{\sigma}_x\rangle_{ss} = \frac{\Gamma_p}{\Gamma_{\rm rel}}\cos(2\phi)$. We verify that the four methods give little difference in the static values of the coherence [see Figs. 8(a) and 8(b)]. On the other hand, we consider the higher-frequency oscillation induced by the driving CR terms. In Figs. 9(a) and 9(b), we show the steady oscillation amplitudes $Q_{1,\infty}$ of the frequency $2\omega_L$ for the four methods. It is notable that the amplitudes for the ζ-ξ_k-TRWA and ζ-TRWA method can be enhanced significantly as the driving strength increases, while those of the other methods, which do not take into account the driving CR terms, are zero. Thus, we point out that the even-frequency

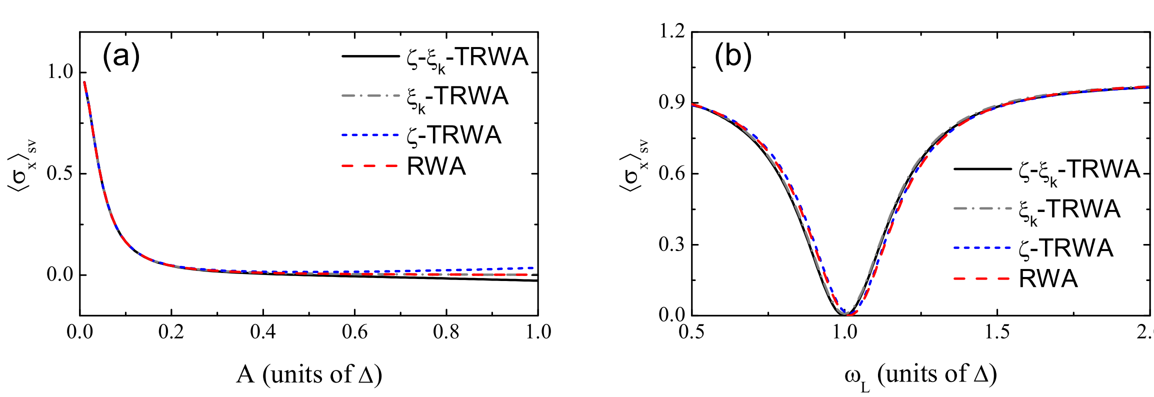

FIG. 8. (Color online) (a) The static value of coherence in the long-time limit as a function of A is shown for $\alpha = 0.01$ and $\omega_L = \Delta$. (b) The static value of coherence as a function of ω_L is shown for $\alpha = 0.01$ and $A = 0.5\Delta$.

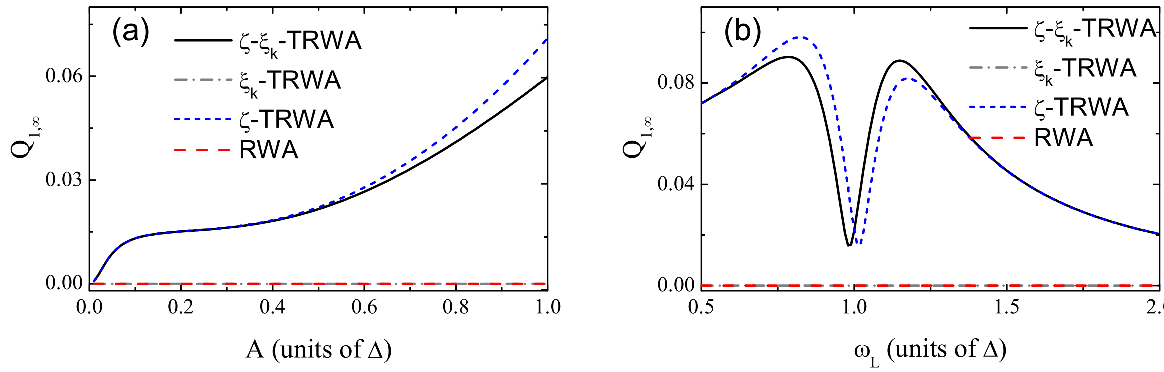

FIG. 9. (Color online) (a) The steady oscillation amplitude of the coherence as a function of A is shown for $\alpha = 0.01$ and $\omega_L = \Delta$. (b) The steady oscillation amplitude of the coherence as a function of ω_L is shown for $\alpha = 0.01$ and $A = 0.5\Delta$.

oscillation is a pure character induced by the driving CR terms. In addition, in Fig. 9(b), the renormalization of the spectral density results in the difference between ζ-ξ_k-TRWA and ξ_k-TRWA for near-resonance cases.

D. Coherent destruction of tunneling

In this section, we apply the ζ-TRWA and ζ-ξ_k-TRWA methods to study how the dissipative environment affects the intriguing phenomenon known as coherent destruction of tunneling (CDT) [29–31]. As discussed in our previous work, the Rabi-RWA method cannot give the CDT under the condition, i.e., $\omega_L = 10\Delta$ and $A = 24.0483\Delta$ [24]. Therefore, the RWA and ξ_k-TRWA methods based on Rabi-RWA cannot give CDT. In Fig. 10, we show the probability of the TLS to remain in its initial state for the CDT condition with dimensionless coupling constant $\alpha = 0.01$. The results of both the ζ-ξ_k-TRWA and ζ-TRWA methods show that the tunneling of the TLS is effectively suppressed. However, the ζ-ξ_k-TRWA method gives a slower dissipative process than that of the ζ-TRWA, which can be understood by analyzing the decay rate $\gamma(\omega)$. Within the description of the ζ-ξ_k-TRWA, we find that the decay rate $\gamma(\omega)$ depends on the renormalized spectral density $\tilde{G}(\omega)$, which has the renormalized factor

$$F = \left[\frac{2J_0\left(\frac{A}{\omega_L}\zeta\right)\eta\Delta}{\omega + J_0\left(\frac{A}{\omega_L}\zeta\right)\eta\Delta}\right]^2$$

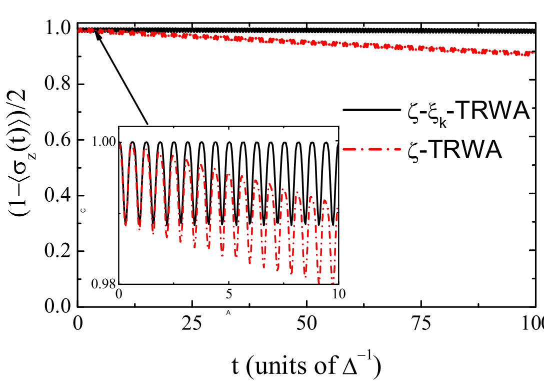

FIG. 10. (Color online) The probability of the TLS remains in its initial state as a function of t is shown for $\alpha = 0.01$, $\omega_L = 10\Delta$, and $A = 24.0483\Delta$.

[see Eq. (19)]. Notice that, under the CDT condition, $J_0(\frac{A}{\omega_L}\zeta)$ tends to zero (not exactly zero) and, thus, the decay rate of the ζ-ξ_k-TRWA method can be significantly reduced, which in turn makes the dissipation effect weaker than that of the ζ-TRWA. Our result indicates that, in spite of the dissipation, it is possible to achieve the CDT and keep the initial state for a relatively long time owing to the interference between driving and dissipation.

V. CONCLUSION

To summarize, we proposed a method based on the unitary transformation to study the dissipative dynamics of the driven spin-boson model. The unitary transformation is applied to construct an effective Hamiltonian, which possesses modifications to the driving- and dissipative-related quantities induced by the effects of the CR terms. Besides, the effective Hamiltonian takes a simple RWA-like form. Starting from the effective Hamiltonian, we are able to derive Born–Markov master equation in the transformed rotating frame, which can be exactly solved without difficulty. The general procedure of the method is simple and clear. Moreover, there are two particular treatments derived from the general treatment based on the generator (7) by setting either $\zeta = 0$ or $\xi_k = 0$ and invoking further the approximation of neglecting the CR terms of the driving or the dissipative coupling, respectively. Including the treatment of neglecting all the CR terms, there are four methods with the effective Hamiltonians of the same mathematical form and corresponding master equations. We systematically examine the results of the four methods from weak to strong driving in the weak-damping regime in order to clarify the effects of the CR terms on the population difference and coherence. We find that the CR terms of the dissipative coupling contribute their influence by renormalizing the spectral density, which characterizes the dissipative property of the bath, and the CR terms of the driving cause the modifications to the detuning and driving strength. In the limit of weak driving and weak damping, we analytically show the asymptotic behavior of the modified quantities, which naturally tend to the bare ones and, thus, the four methods give consistent results. In the regime of strong driving and weak damping, the four methods generally give different results owing to the effects of CR terms. In the case of resonant strong driving, we demonstrated that the CR terms of both the driving and dissipative coupling prolong the relaxation and dephasing processes and significantly influence the steady state of the TLS. In the case of largely detuned driving, it turned out that the influence of the CR terms of the dissipative coupling becomes negligible while the CR terms of the driving contribute the dominant modifications to the steady-state solutions. In addition, we applied the methods of taking account of the CR terms of the driving to study how the dissipation influences the CDT. The general ζ-ξ_k-TRWA treatment illustrates that, in the presence of the CR terms of both the driving and dissipative coupling, the system is able to survive in its initial state under the CDT condition for a relatively long time without obvious decay.

We identify the relation between our method and the FBM approach by the comparison of the results of the ζ-TRWA + SA method with those of the FBM approach. In the valid parameter space of our method for nondissipative driven dynamics, it is interesting to find the numerical equivalence between the FBM approach and the ζ-TRWA + SA solution. The equivalence reflects that the role of MRWA invoked in the FBM approach is approximately equivalent to two approximations in our treatment: (i) the neglect of $\frac{1}{2}\sum_k g_k(b_k^\dagger\sigma_+ + b_k\sigma_-)$ in the transformed Hamiltonian and (ii) the secular approximation. To our knowledge, the FBM and the ζ-TRWA approaches properly take into account the effects of the driving CR terms but do not consider the contributions of the dissipative CR terms because of the use of the MRWA in the FBM approach and the neglect of $\frac{1}{2}\sum_k g_k(b_k^\dagger\sigma_+ + b_k\sigma_-)$ in the ζ-TRWA treatment, respectively. In the Born–Markov formalism, these method are sufficient to describe the issue when the driving is largely detuned in which case the renormalization of the spectral density becomes unobservable. However, in the case of the resonant strong driving, we must take into account the effects of the dissipative CR terms, which causes the renormalization of the spectral density in comparison with those of the RWA by using the bare spectral density and leads to a different dynamical evolution of the system.

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